

COMPARISON OF TWO IMMIGRATION SCHEMES ON  
THE SURVIVAL RATE IN A COLONY

BU-165-M

Keewhan Choi

April, 1964

Abstract

Using a very simple model, the survival rates of an individual in a colony are compared under two different immigration schemes. It is found that if life at the home territory is easier than or the same as that of the colony, the survival rate is higher under the exodus scheme of immigration. Otherwise, the survival rate is higher under the trickle scheme.

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Summary

Using a very simple model, the survival rates of an individual in a colony are compared under two different immigration schemes. It is found that if life at the home territory is easier than or the same as that of the colony, the survival rate is higher under the exodus scheme of immigration. Otherwise, the survival rate is higher under the trickle scheme.

The following problem was given in the general ecology class of Professor D. Pimentel.

For some species of animals, animals regularly leave their home territory and establish a colony. The question of interest is the survival rate of an individual in a colony under each of the two schemes of immigration (to a colony). The first one is an exodus scheme, i.e., one half of the population leaves home territory every 11 years for a colony. The second one is a trickle scheme, whereby a 5% of the population strikes out for a colony every year.

The following simple model is used for the comparison of survival rates (of an individual). Yearly survival rate at the home territory or in the colony is assumed to be independent and constant. Let  $P_H$  and  $P_C$  be the probability of yearly survival at the home territory or the colony respectively. Let  $P_i(K)$  be the probability of immigrating under scheme  $i$  to a colony during the first 11 years and surviving at least until  $(11 + K)^{th}$  year, where  $i$  stands for either E (exodus) or T (trickle). The survival rates in a colony under two schemes are compared using  $P_E(K)$  and  $P_T(K)$ .

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$P_E(K)$  and  $P_T(K)$  are obtained as follows:

$$P_E(K) = 0.5 P_H^{11} P_C^K$$

$$\begin{aligned} P_T(K) &= 0.05 \sum_{i=0}^{11} P_H^i P_C^{K+11-i} \\ &= 0.05 P_C^K \sum_{i=0}^{11} P_H^i P_C^{11-i} \end{aligned}$$

Let 
$$S = \sum_{i=0}^{11} P_H^i P_C^{11-i} .$$

$$\text{If } P_C > P_H, \quad S = P_C^{11} \frac{1 - \left(\frac{P_H}{P_C}\right)^{12}}{1 - \frac{P_H}{P_C}} \approx \frac{P_C^{12}}{P_C - P_H} .$$

$$\text{If } P_C < P_H, \quad S = P_H^{11} \frac{1 - \left(\frac{P_C}{P_H}\right)^{12}}{1 - \left(\frac{P_C}{P_H}\right)} \approx \frac{P_H^{12}}{P_H - P_C} .$$

$$\text{If } P_C = P_H = P, \quad S = 12P^{11} .$$

Let us now compare  $P_E(K)$  and  $P_T(K)$  .

(i) If  $P_C > P_H$ , then  $P_E(K) < P_T(K)$  .

Proof

$$P_E(K) < P_T(K) \Leftrightarrow .5 P_H^{11} < .05 \frac{P_C^{12}}{P_C - P_H}$$

$$\Leftrightarrow 10 \left( P_H^{11} P_C - P_H^{12} \right) < P_C^{12}$$

$$\Leftrightarrow 10 \left( \left( \frac{P_H}{P_C} \right)^{11} - \left( \frac{P_H}{P_C} \right)^{12} \right) < 1$$

$$\Leftrightarrow 10 \left( \frac{P_H}{P_C} \right)^{11} \left( 1 - \frac{P_H}{P_C} \right) < 1$$

$$\Leftrightarrow 1 - \frac{P_H}{P_C} < 1.$$

(ii) If  $P_C < P_H$ , then  $P_T(K) < P_E(K)$ .

Proof Similar to case (i).

(iii) If  $P_C = P_H = P$ , then  $P_T(K) > P_E(K)$ .

Proof  $0.5 P^{11} < (0.05)^{12} P^{11}$ .

Note that  $\left( \frac{P_C}{P_H} \right)^{12}$  and  $\left( \frac{P_H}{P_C} \right)^{12}$  are taken to be 0 in cases (i) and (ii).

If the home life is easier than or the same as the colony life, (cases (ii) and (iii)), the survival rate in the colony is higher under the exodus scheme. If the colony life is easier than the home life (case (i)), the survival rate in the colony is higher under the trickle scheme.

Comments: The above analysis is too simple to be of much practical value. What is required in the next stage of analysis is to set up a difference-differential equations satisfied by the probability of  $x$  animals at time  $P_x(t)$  and under two immigration schemes. More specifically, take two immigration rates (one of which has the period of 11 years and the other one year) and compare  $P_x(t)$  under the two rates. Even for the case of linear birth and linear death rates, I could not obtain the solution to the difference-differential equations. Comparison of the numerical solution might be of some interest.